

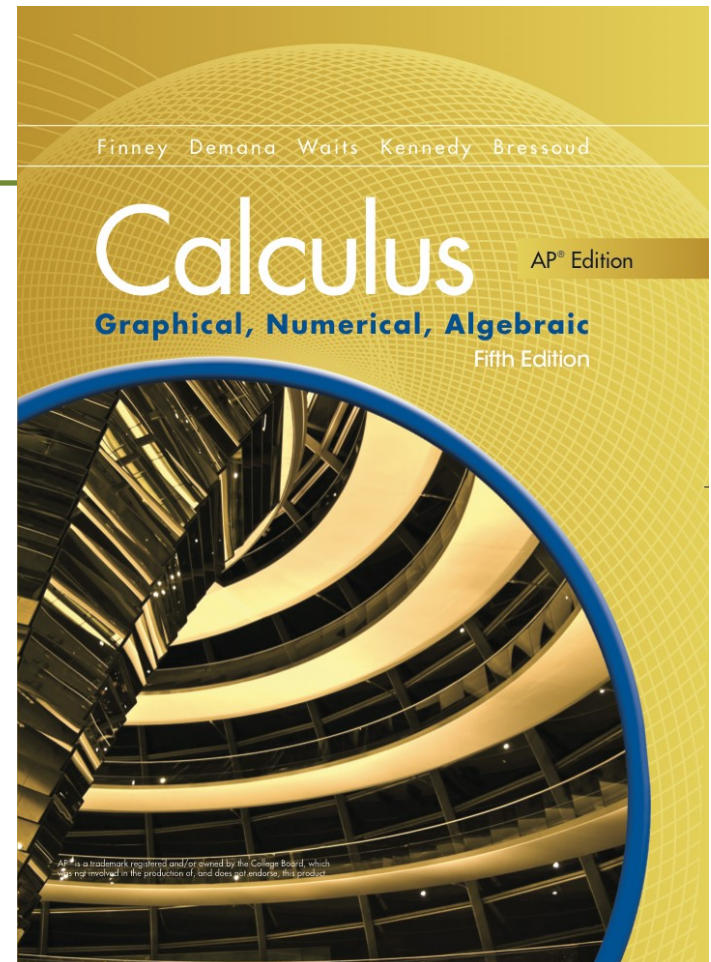
Chapter 5

Applications of Derivatives



Section 5.2

Mean Value Theorem



Quick Review

Find exact solutions to the inequality.

1. $2x^2 - 4 < 0$
2. $4x^2 - 4 > 0$

Let $f(x) = \sqrt{6 - 2x^2}$.

3. Find the domain of f .
 4. Where is f continuous?
 5. Where is f differentiable?
-
6. Given $f(x) = x^2 + 3x + C$. Find C so that the graph of the function f passes through the point $(1, -1)$.

Quick Review Solutions

Find exact solutions to the inequality.

1. $2x^2 - 4 < 0$ $(-\sqrt{2}, \sqrt{2})$

2. $4x^2 - 4 > 0$ $(-\infty, -1) \cup (1, \infty)$

Let $f(x) = \sqrt{6 - 2x^2}$.

3. Find the domain of f . $[-\sqrt{3}, \sqrt{3}]$

4. Where is f continuous? $[-\sqrt{3}, \sqrt{3}]$

5. Where is f differentiable? $(-\sqrt{3}, \sqrt{3})$

6. Given $f(x) = x^2 + 3x + C$. Find C so that the graph of the function f passes through the point $(1, -1)$. $C = -5$

What you'll learn about

- The Mean Value Theorem
- Increasing and decreasing functions
- Functions whose derivative is zero
- Antidifferentiation

...and why

The Mean Value Theorem is an important theoretical tool to connect the average and instantaneous rates of change.

Mean Value Theorem for Derivatives

If $y = f(x)$ is continuous at every point of the closed interval $[a, b]$ and differentiable at every point of its interior (a, b) , then there is at least one point c in (a, b) at which the instantaneous rate of change equals the mean rate of change,

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Example Explore the Mean Value Theorem

Show that the function $f(x) = x^2$ satisfies the hypothesis of the Mean Value Theorem on the interval $[0, 2]$. Then find a solution c to the equation

$$f'(c) = \frac{f(b) - f(a)}{b - a} \text{ on this interval.}$$

The function $f(x) = x^2$ is continuous on $[0, 2]$ and differentiable on $(0, 2)$.

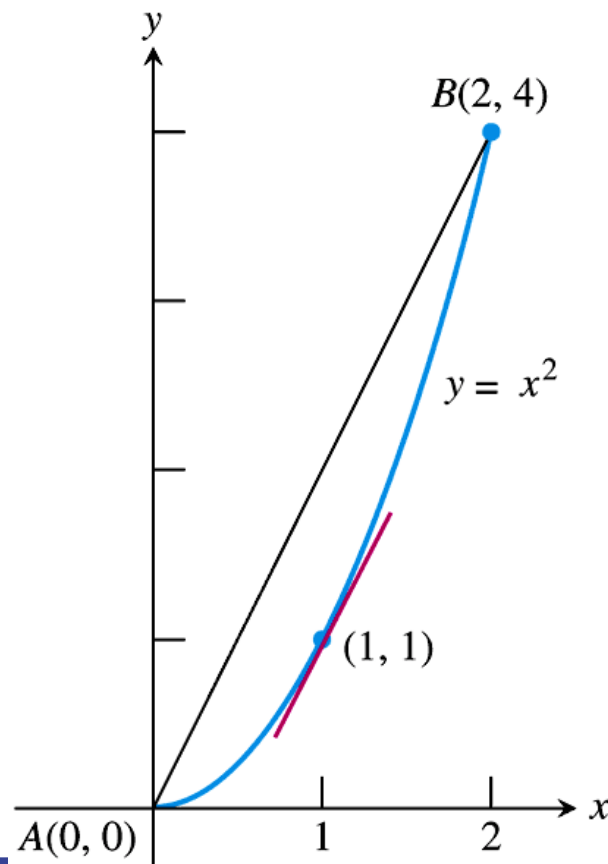
Since $f(0) = 0$, $f(2) = 4$, and $f'(x) = 2x$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$2c = \frac{4 - 0}{2 - 0}$$

$$2c = 2$$

$$c = 1.$$



Increasing Function, Decreasing Function

Let f be a function defined on an interval I and let x_1 and x_2 be any two points in I .

1. f **increases** on I if $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$.
2. f **decreases** on I if $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$.

Corollary: Increasing and Decreasing Functions

Let f be continuous on $[a, b]$ and differentiable on (a, b) .

1. If $f' > 0$ at each point of (a, b) , then f increases on $[a, b]$.
2. If $f' < 0$ at each point of (a, b) , then f decreases on $[a, b]$.

Example Determining Where Graphs Rise or Fall

Where is the function $f(x) = 2x^3 - 12x$ increasing and where is it decreasing?

The function is increasing when $f'(x) > 0$.

$$6x^2 - 12 > 0$$

$$x^2 > 2$$

$$x < -\sqrt{2} \text{ or } x > \sqrt{2}$$

The function is decreasing when $f'(x) < 0$.

$$6x^2 - 12 < 0$$

$$x^2 < 2$$

$$-\sqrt{2} < x < \sqrt{2}$$

Corollary: Functions with $f' = 0$ are Constant

If $f'(x) = 0$ at each point of an interval I , then there is a constant C for which $f(x) = C$ for all x in I .

Corollary: Functions with the Same Derivative Differ by a Constant

If $f'(x) = g'(x)$ at each point of an interval I , then there is a constant C such that $f(x) = g(x) + C$ for all x in I .

Antiderivative

A function $F(x)$ is an **antiderivative** of a function $f(x)$ if $F'(x) = f(x)$ for all x in the domain of f .

The process of finding an antiderivative is **antidifferentiation**.

Example Finding Velocity and Position

Find the velocity and position functions of a freely falling body for the following set of conditions:

The acceleration is 9.8 m/sec^2 and the body falls from rest.

Assume that the body is released at time $t = 0$.

Velocity: We know that $a(t) = 9.8$ and $v(0) = 0$, so

$$v(t) = 9.8t + C$$

$$v(0) = 0 + C$$

$C = 0$. The velocity function is $v(t) = 9.8t$.

Position: We know that $v(t) = 9.8t$ and $s(0) = 0$, so

$$s(t) = 4.9t^2 + C$$

$$s(0) = 0 + C$$

$C = 0$. The position function is $s(t) = 4.9t^2$.